MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

DEM5028 – ENGINEERING MATHEMATICS 2

(Diploma in Electronic Engineering)

4 MARCH 2019

2.30 PM – 4.30 PM

(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **3** pages (excluding the cover page and appendices).
2. Attempt **ALL FOUR** questions.
3. Write your answers in the answer booklet provided.
4. Formulae are provided in the appendix section.

QUESTION 1

a. Evaluate the following integrals.

i. $\int \frac{5x^2+20x+6}{x^3+2x^2+x}dx$ (8 marks)

ii. $\int_0^{\frac{\pi}{4}} x\sec^2 x \; dx$ (5 marks)

b. The Diagram 1 below shows a straight line $2x+y=16$ intersecting a curve $y=(x-4)^2$ at point P and point Q. Find

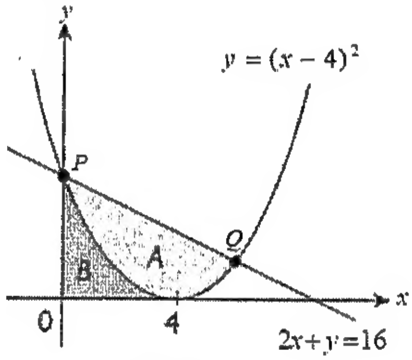

Diagram 1

i. the area of the region A. (8 marks)

ii. the volume generated when the shaded region B is revolved through $360°$ about the x-axis. (4 marks)

[TOTAL 25 MARKS]

Continued ...

QUESTION 2

a. Find the general solution of the differential equation $2+\dfrac{dy}{dx}=\dfrac{3x}{y-2}+2$. (5 marks)

b. Find the solution of the differential equation $y^2\dfrac{dy}{dx}=-3y\sqrt{x}$, given that $y=1$ when $x=1$. (9 marks)

c. Find the solution of differential equation $x\dfrac{dy}{dx}-x^4=-y$, subject to $y(1)=1$.

(11 marks)

[TOTAL 25 MARKS]

QUESTION 3

a. Find the sum to infinity for the series (4 marks)

$$18-6+2-\ldots$$

b. Expand $(9+x)^{\frac{1}{2}}$ in ascending powers of x up to the term in x^2. (7 marks)

c. Given that $A(3,-1,1), B(-1,2,-2)$ and $C(3,2,-2)$ are three vertices of a triangle

   i. Find $\overrightarrow{AB}\times\overrightarrow{BC}$. (7 marks)

   ii. Calculate the area of triangle ABC. (7 marks)

[TOTAL 25 MARKS]

Continued ...

QUESTION 4

a. If $f(x,y) = e^{2x}\sin(3x+2y)$.

i. Compute $\left.\dfrac{\partial f}{\partial x}\right|_{\left(0,\frac{1}{8}\pi\right)}$ and $\left.\dfrac{\partial f}{\partial y}\right|_{\left(0,\frac{1}{8}\pi\right)}$. (9 marks)

ii. Evaluate f_{xx} and f_{yy}. (8 marks)

b. Find the volume of the solid bounded above by the plane $z = 4 - x - y$ and below by the rectangle $R = \{(x,y) : 0 \le x \le 1,\ 0 \le y \le 2\}$. (8 marks)

[TOTAL 25 MARKS]

End of Page.

APPENDICES: Formulae

Integration of common functions

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ $\int \frac{1}{x} dx = \ln|x| + C$ $\int e^x dx = e^x + C$ $\int a^x dx = \frac{a^x}{\ln a} + C$

$\int \sin x dx = -\cos x + C$ $\int \cos x\, dx = \sin x + C$ $\int \sec^2 x\, dx = \tan x + C$ $\int \sec x \tan x dx = \sec x + C$

$\int \csc^2 x dx = -\cot x + C$ $\int \csc x \cot x dx = -\csc x + C$

Inverse Trigonometry

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \cot^2 x = \csc^2 x$$
$$1 + \tan^2 x = \sec^2 x$$

Integration by parts

$$\int u dv = uv - \int v du$$

Areas Between Curves

$$A = \int_a^b [f(x) - g(x)] dx$$
$$A = \int_c^d [w(y) - v(y)] dy$$

Volume by Washer

$$V = \int_a^b \pi\left([f(x)]^2 - [g(x)]^2\right) dx$$
$$V = \int_c^d \pi\left([w(y)]^2 - [v(y)]^2\right) dy$$

Volume by Cylindrical Shells

$$V = \int_a^b 2\pi x\left(f(x) - g(x)\right) dx$$
$$V = \int_c^d 2\pi y\left(w(y) - v(y)\right) dy$$

Linear Differential Equations:

$$\frac{dy}{dx} + p(x)y = q(x);\ \mu y = \int \mu q(x) dx \Rightarrow y = \frac{1}{\mu}\int \mu q(x) dx, \qquad \text{where } \mu = e^{\int p(x)dx}$$

Divergence Test	If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.
p-series	The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if p > 1 and divergent if $p \leq 1$.
Limit Comparison Test	Let $\sum a_n$ and $\sum b_n$ be series with positive terms such that $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ If $0 < c < \infty$, then both series converge or both diverge.
Alternating Series Test	If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \ldots \qquad b_n > 0$ Satisfies : i. $b_{n+1} \leq b_n$ for all n ii. $\lim_{n\to\infty} b_n = 0$ then the series is convergent
Ratio Test	Let $\sum a_n$ be a series with nonzero terms such that $L = \lim_{n\to\infty} \frac{\lvert a_{n+1}\rvert}{\lvert a_n\rvert}$ a. Series converges absolutely if $L < 1$ b. Series diverges if $L > 1$ or $L = \infty$ c. No conclusion if $L = 1$

Binomial expansion

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Vector

The length of the vector **a** $=< a_1, a_2, a_3 >$ is $|a| = \sqrt{a_1^{\ 2} + a_2^{\ 2} + a_3^{\ 2}}$.

If θ is the angle between the vector **a** and **b**, then **a.b** $= |\mathbf{a}||\mathbf{b}|\cos\theta$ & $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|$ $\sin\theta$

Cross Product

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

Equation of Line

Vector equation: $\boldsymbol{r} = \boldsymbol{r_0} + t\boldsymbol{v}$

Parametric equation: $\boldsymbol{x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct}$

Equation of Plane $< a, b, c >< x - x_0, y - y_0, z - z_0 >= 0$

The Chain Rule

Suppose that $z = f(x, y)$, where $x = g(t)$ and $y = h(t)$ $\Rightarrow$ $\dfrac{dz}{dt} = \dfrac{\partial f}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial f}{\partial y}\dfrac{dy}{dt}$

Second Derivatives Test

Suppose that $f_x(a,b) = 0$ and $f_y(a,b) = 0$ [that is, (a, b) is a critical point of f]. Let $D = D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$

a. If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a, b)$ is a local minimum.

b. If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a, b)$ is a local maximum

c. If $D < 0$, then $f(a, b)$ is a saddle point.

Moments and Centers of Mass

The **moment** about the ***x*-axis:**

$$M_x = \iint_D y\rho(x,y)\,dA$$

The **moment** about the **y-axis:**

$$M_y = \iint_D x\rho(x,y)\,dA$$

The coordinates $(\bar{x}, \bar{y})$ of the center of mass:

$$\bar{x} = \frac{M_y}{m} = \frac{1}{m}\iint_D x\rho(x,y)\,dA \qquad \bar{y} = \frac{M_x}{m} = \frac{1}{m}\iint_D y\rho(x,y)\,dA$$

Where the mass:

$$m = \iint_D \rho(x,y)\,dA$$

Triple Integrals: $\displaystyle\iiint_B f(x,y,z)\,dV = \int_r^s \int_c^d \int_a^b f(x,y,z)\,dx\,dy\,dz$